

Adler function from $R^{e^+e^-}(s)$ measurements: experiments vs QCD theory.

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An experimentally motivated QCD analysis of the behaviour of the Adler D -function in the Euclidian region is described. It is stressed that by taking account of b -quark mass-dependent α_s^2 -effects one obtains better agreement between theoretical predictions and experimentally motivated behaviour of the D -function at large Euclidian momentum transfer. A more detailed analysis of QCD predictions, including information on quark and gluon condensates, requires more precise data on $e^+e^- \rightarrow \text{hadrons}$, particularly in the energy regions $E < M_{J/\Psi}$ and $M_{J/\Psi} < E < 3.6$ GeV. Use of experimental determination of the D -function to test the generalized Crewther relation is outlined.

1. Introduction. The process e^+e^- -annihilation into hadrons is one of the most informative processes in elementary particle physics. Over the past few decades special attention has been paid to detailed theoretical and experimental study of its basic characteristics, and in particular, of the ratio $R^{e^+e^-}(s) = \sigma_{tot}(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$. As a result, important information about the properties of hadrons and of their constituents (quarks and gluons) was obtained. For example, e^+e^- collisions enabled discovery in Novosibirsk of the first direct evidence for the ρ^0 -meson [1]. The J/Ψ resonance [2] was simultaneously discovered by the e^+e^- -collider SPEAR at SLAC, and at BNL as a result of study of the process $p + p \rightarrow e^+e^- + X$ [3]. Rapid confirmation came from a slight increase of the beam energy of the ADONE e^+e^- collider [4].

The observation of Ψ' and Ψ'' particles and their interpretation as $\bar{c}c$ bound states of quarks (e.g. see the review [5]) has served as an incitement to the development of the theory of strong interactions and QCD in particular. It brought up the question of the possibility of finding the signal for the fifth quark in e^+e^- -annihilation. Moreover, an indirect determination of the mass

of the sixth t -quark was made from detailed fits of the experimental data of the high-energy e^+e^- -colliders LEP and SLC at the Z^0 -pole taking into account the effects of its virtual propagation (for a review see [6]). Impressively, the extracted mass of the t -quark turned out to be in agreement with the its direct "measurement", resulting from subsequent discovery of the 6th quark at the Tevatron.

Among present theoretical studies of data from e^+e^- -colliders data are attempts to find the phenomenologically allowed window for the mass of the Standard Model Higgs boson. These studies are based on combined fits of data from the high-energy colliders LEP and SLC (for a review see [7]) taking into account the effect of running of the inverse electromagnetic coupling constant from its low-energy value $\alpha^{-1}(0) \approx 137.0356\dots$ to M_Z . At this reference scale its value was determined using a compilation of the available e^+e^- -data [8]. The result of Ref.[8] $\alpha^{-1}(M_Z) = 128.896(90)$ was recently updated to give $\alpha^{-1}(M_Z) = 128.913(35)$ [9] (for a discussion of the fast developing situation in this area see the review of Ref.[10] and other related works on the subject [11]). Further estimates of experimental and theoretical uncertainties in this important quantity are on the agenda.

2. Electron-positron annihilation: experiment vs QCD predictions. From the perspective of the scanned energy regions, and the physical problems under investigation, all past,

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Table 1
Classification of constructed and planned e^+e^- -colliding machines

Group	Machine	Location	Start Data	Status in 1999 June	Beam Energy (b.e.) $E_{b.e.} + E_{b.e.}$ [GeV]	Region
I	AdA	Frascati	1960	—		
		ORSAY	1961	—		0.2 + 0.2
	VEPP-2	Novosibirsk	1966	—	0.2–0.55 + 0.2–0.55	
	ACO	ORSAY	1966	—	0.2–0.55 + 0.2–0.55	
	ADONE	Frascati	1969	—	0.7–1.55 + 0.7–1.55	
	VEPP-2M	Novosibirsk	1974	working	0.2–0.67 + 0.2–0.67	
	DCI	ORSAY	1976	—	0.5–1.7 + 0.5–1.77	
	ϕ -factory					
	DAΦNE	Frascati	1999	launched		0.51+ 0.51
II	CEA	Cambridge (USA)	1971	—	1.5–3.5 + 1.5–3.5	
	SPEAR	SLAC	1972	—	1.2–4.2 + 1.2–4.2	
	DORIS	Hamburg	1974	—	1–5.1 + 1–5.1	
	VEPP-4	Novosibirsk	1979	planned to restart	1.5–5 + 1.5–5	
	CESR	Cornell	1979	working	3–8 + 3–8	
	BEPC	Bejing	1991	working	1–2.5 + 1–2.5	
	c - τ -factory	Bejing	>1999	was planned	1.5–3 + 1.5–3	
III	B-factory			asymmetric colliders		
	PEP-II	SLAC	1999	launched		9 + 3.1
	B-factory	KEK	1999	launched		8 + 3.5
IV	PETRA	Hamburg	1978	—	5–19 + 5–19	
	PEP	Stanford	1980	—	5–18 + 5–18	
	TRISTAN	KEK	1987	—	25–30 + 25–30	
V	SLC	SLAC	1989	working	45–50 + 45–50	
	LEP-I	CERN	1989	—	45–47 + 45–47	
	LEP-II	CERN	1995	working	65–100 + 65–100	
VI	TESLA	DESY	> 2005	linear colliders	250–500 + 250–500	
	NLC	SLAC	> 2005	(projects under	250–500 + 250–500	
	JLC	KEK	> 2005	discussions)	250–500 + 250–500	

present and proposed e^+e^- -colliders may be divided into six groups. We present this classification in Table 1, updating the material given in Ref.[12]. The new information was gained, in part, from material in Ref.[13].

These colliders provide complementary information about the behaviour of the $e^+e^- \rightarrow \text{hadrons}$ total cross-section, at different energies, from machines of different luminosity. Moreover, some important physical problems under investigation necessitate more precise experimental data, not only at high energies, but also in low and intermediate energy regions. Indeed, the study of the latter regions, both by experimen-

tal and theoretical methods, may provide better estimates of

$$\Delta\alpha_{had}(q^2) = -\frac{\alpha q^2}{3\pi} \text{Re} \int_{4m_\pi^2}^{\infty} ds \frac{R^{e^+e^-}(s)}{s(s-q^2-i\epsilon)} \quad (1)$$

which is the hadronic vacuum-polarization contribution to the value of the effective fine structure constant.

Another important characteristic of the process e^+e^- -hadrons was introduced in Ref.[14]. It concerns the Euclidian Adler D -function

$$D(Q^2) = Q^2 \int_{4m_\pi^2}^{\infty} \frac{R^{e^+e^-}(s)}{(s+Q^2)^2} ds \quad (2)$$

which can be related to Eq.(1) as follows:

$$D(-s) = \frac{3\pi}{\alpha} s \frac{d}{ds} \Delta\alpha_{had}(s) \quad (3)$$

The study of this quantity has a number of attractive features. Its behaviour was first analyzed from an experimental point of view some time ago [14, 15]. In the recent work of Ref.[16] this problem was reconsidered using a compilation of e^+e^- experimental data [8]³. The results are depicted in Fig.1 and Fig.2 where the shaded areas represent the $\pm 1\sigma$ band obtained from the data.

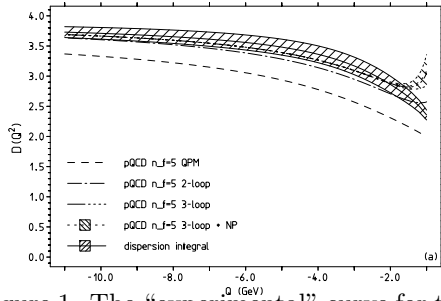


Figure 1. The “experimental” curve for the Adler function together with QCD predictions

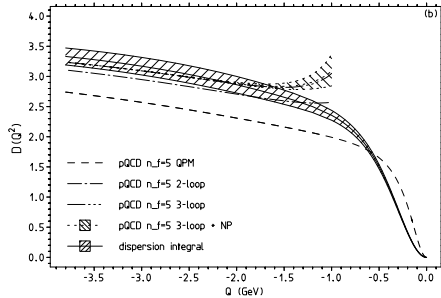


Figure 2. The low energy “experimental” and QCD behaviour of the Adler function

It is interesting to compare the “experimental” behaviour for the D -function with its QCD theoretical expression. One can express it in the following form

$$D^{QCD}(Q^2) = D^{PT}(Q^2) + D^{NP}(Q^2) \quad (4)$$

where the first part describes the perturbative QCD contributions, while the second term takes

³It would be interesting to include in this and related analyses the new measurements of $R^{e^+e^-}(s)$ at \sqrt{s} values of 2.6, 3.2, 3.4, 3.55, 4.6 and 5 GeV, from BEPC [17].

account of higher-twist effects, which are related to the vacuum condensates of quark and gluon fields [18]. In the work of Ref.[16] we considered an α_s^2 mass-dependent expression for $D^{PT}(Q^2)$, namely

$$D^{PT}(Q^2) = D^{(0)}(Q^2) + D^{(1)}(Q^2) \frac{\alpha_s(Q^2)}{\pi} + D^{(2)}(Q^2) \left(\frac{\alpha_s(Q^2)}{\pi} \right)^2 \quad (5)$$

The Born term expression is well known and has the following form

$$D^{(0)}(Q^2) = 3 \sum_f Q_f^2 \left(1 - 6x + \frac{12x^2}{\sqrt{1+4x^2}} \ln \left[\frac{\sqrt{1+4x} + 1}{\sqrt{1+4x} - 1} \right] \right) \quad (6)$$

where $x = m_f^2/Q^2$, while m_f is the pole quark mass. The analytic mass-dependent expression for $D^{(1)}$ can be obtained from the calculations of Ref.[19], which are in agreement with the results of Ref.[20]. Since the mass dependence of the 3-loop term $D^{(2)}$ -term in Eq.(5) is not yet known analytically, detailed information about its small-mass [21, 22] and heavy-mass [23] expansions was used in Ref.[16] to approximate the behaviour of $D^{(2)}$ in the Euclidian region, with high precision, using conformal mapping and Padé improvement [24]. However, the theoretical expression for mass dependence of the double-bubble three-loop vacuum polarization diagram was not included in the theoretical expression for $D^{(2)}$ -term in Ref.[16]. There are arguments, based on the calculations of $(m^2/Q^2)\alpha_s^2$ -corrections [21], that this contribution is small. Let us consider the non-perturbative contributions to $D^{NP}(Q^2)$. In Ref.[16] the following expression was used

$$D^{NP}(Q^2) = 3 \sum_f Q_f^2 (8\pi^2) \times \left[\frac{1}{12} \left(1 - \frac{11}{18} \frac{\alpha_s(\mu^2)}{\pi} \right) \frac{\langle \frac{\alpha_s}{\pi} G^2 \rangle}{Q^2} + 2 \left(1 + \frac{\alpha_s(\mu^2)}{3\pi} + O(\alpha_s^2) \right) \frac{\langle m_q \bar{q}q \rangle}{Q^4} + \left(\frac{4}{27} \frac{\alpha_s(\mu^2)}{\pi} + O(\alpha_s^2) \right) \sum_{q'} \frac{\langle m_{q'} \bar{q}'q' \rangle}{Q^4} \right] \quad (7)$$

Table 2

The origin of uncertainties for the “experimental” Adler function at $Q = 2.5$ GeV and M_Z

	$D(2.5 \text{ GeV})$	rel. err.	abs. err.	$D(M_Z)$	rel. err.	abs. err.
Resonances	.688 (.025)	3.6 %	0.8 %	.004 (.000)	5.2 %	0.0 %
$E < M_{J/\Psi}$	1.068 (.127)	11.9 %	4.2 %	.002 (.000)	14.9 %	0.0 %
$M_{J/\Psi} < E < 3.6 \text{ GeV}$.178 (.035)	19.9 %	1.2 %	.001 (.000)	19.8 %	0.0 %
$3.6 \text{ GeV} < E < M_\Upsilon$.850 (.055)	6.4 %	1.8 %	.032 (.002)	7.0 %	0.1 %
$M_\Upsilon < E < 12 \text{ GeV}$.088 (.008)	8.7 %	0.3 %	.024 (.002)	9.0 %	0.1 %
$E < 12 \text{ GeV}$ data	2.871 (.146)	5.1 %	4.8 %	.063 (.003)	4.9 %	0.1 %
$12 \text{ GeV} < E$ QCD	.162 (.001)	0.3 %	0.0 %	3.755 (.006)	0.2 %	0.2 %
total	3.033 (.146)	4.8 %	4.8 %	3.818 (.007)	0.2 %	0.2 %

The terms beyond leading order in α_s , calculated in the works of Ref.[25], turned out to be not so important at the moment, because the current experimental data for the D -function gives only rough constraints on the contributions of dimension-four condensates, which were varied in Ref.[16] within the following intervals $< \frac{\alpha_s}{\pi} G^2 > \approx (0.336 - 0.442 \text{ GeV})^4$, $< m_u \bar{u}u > = < m_d \bar{d}d > = -(0.086 - 0.111 \text{ GeV})^4$, $< m_s \bar{s}s > = -(0.192 - 0.245 \text{ GeV})^4$. Nevertheless, in Fig.1 and Fig.2 one can see the region, where the addition of the nonperturbative QCD corrections to the three-loop perturbative QCD expression leads to deviation from the experimentally allowed region for the D -function at low Euclidian momentum transfer, Q^2 . It is possible, that taking into account higher-order power corrections [26] might improve the agreement with “experimental data” at low energies, shown in Fig.2. Moreover, it is of real interest to update the previous analysis, in Refs.[27, 28], of the low-energy e^+e^- experimental data (see Refs.[27, 28]) with the help of the QCD Borel sum rule method [18]

$$\int_0^\infty R^{th}(s) e^{-s/M^2} ds = \int_0^\infty R^{e^+e^-}(s) e^{-s/M^2} ds \quad (8)$$

In the process of such an analysis, new low-energy experimental data, obtained in part in Novosibirsk, can be used. In the theoretical part of sum rule of Eq.(8) one should also include perturbative contributions to the coefficients function of quark and gluon condensates of dimension four [25], available from the results of Ref.[26], higher-dimension condensates, and information about massless α_s^3 contributions to $R^{th}(s)$

[29]. As to the sum-rule analysis of the high-energy e^+e^- experimental data, one can try to update the studies of Ref.[30], performed with the help of the finite energy sum rules approach

$$\int_0^{s_0} R^{th}(s) ds = \int_0^{s_0} R^{e^+e^-}(s) ds \quad (9)$$

Concerning the curves for the D -function extracted from experimental data (see Figs. 1,2 of Ref.[16]), it is worth noting that the theoretical analyses of the recent works [31, 32] result in a description of the low-energy tail of Fig.2 by completely different ideas, related to the concept of “freezing” of the QCD coupling constant at small energies [33, 34]. These ideas have realizations, distinct from conclusions in Ref.[35], based on application of the PMS approach [36] to the four-loop massless theoretical expression for $R^{e^+e^-}(s)$. Indeed, it was argued in Ref.[37], that the observed “perturbative” freezing may be spurious, indicating breakdown of a next-to-next-to-leading PMS expression, around the scale of ρ -meson mass. Note, however, that the low-energy results for the fit of e^+e^- data on $R^{e^+e^-}(s)$, performed in Ref.[35] with the help of the following sum rule [38]

$$\bar{R}(s, \Delta) = \frac{\Delta}{\pi} \int_0^\infty \frac{R^{e^+e^-}(s')}{(s' - s)^2 + \Delta^2} ds' \quad (10)$$

merited serious attention. It should be stressed that the low-energy Novosibirsk data of Ref.[39] turned out to be essential in this analysis. For example, they served as an ingredient in the phenomenological part of the considerations of Ref.[40], devoted to the analysis of “analytical” freezing of the QCD coupling constant α_s

in the Minkowskian region, and also in Ref.[41], devoted to consideration of the Crewther relation [42] and its \overline{MS} -scheme QCD generalization [43], using commensurate scale relations [44] (for reviews see [45, 46]). It should be recalled that the α_s^3 -generalization of the Crewther relation connects a massless α_s^3 theoretical expression for the e^+e^- -annihilation D -function [29] with the massless theoretical expressions for the Gross-Llewellyn Smith sum rule of the νN deep-inelastic scattering and the Bjorken sum rule of polarized deep-inelastic scattering, which were calculated at the α_s^3 -level in Ref.[47]. This connection involves the first and second terms of the two-loop approximation to the QCD β -function. We will consider this problem in more detail in the next Section.

We now return to the results of Ref.[16]. Before discussing the comparison of the three-loop massive theoretical QCD predictions for the D -function with the “experimental” behaviour of the D -function at higher momentum transfer, it is worth mentioning that in the process of extracting the “experimental” behaviour of the D -function not all e^+e^- data up to 40 GeV were applied. Indeed, in Ref.[16] a conservative attitude was adopted: the real data were replaced by perturbative QCD results in certain regions, but only where it was obviously safe to do so. Thus, in the regions from 4.5 GeV to M_Υ and above 12 GeV perturbative QCD results were used, including massive three-loop [22] and a massless four-loop QCD contribution [29].

The origins of uncertainty in the “experimental” curves for the D -function were analyzed in Ref.[16] (see Table 2). The main sources come from the region $E < M_{J/\Psi}$, **accessible for more detailed experimental inspection at VEPP-2M and BEPC**, and the region $M_{J/\Psi} < E < 3.6$ GeV, **which is the privilege of BEPC, VEPP-4** and a possible future $c - \tau$ factory. However, as was shown in Ref.[16], even at the current level of experimental precision one can already obtain new information, namely a demonstration of the importance of two-loop heavy-quark mass-dependent effects for the “experimental” behaviour of the D -function at moderate and high Euclidian mo-

mentum transfers. Indeed, after including mass effects, both in the three-loop perturbative part of the D -function, and also in the two-loop running of the QCD coupling constant α_s from the value of $\alpha_{s,\overline{MS}}(M_Z) = 0.120 \pm 0.003$ to lower energy scales, via a variant in Ref.[48] of the momentum ⁴ subtraction scheme, one can observe the appearance of real agreement of the three-loop massive theoretical expression for the D -function with the experimentally-motivated Euclidian curves of Fig.1 and Fig.2. It should be stressed, that if we had not included the three-loop massive term, a discrepancy with the “experimental” behaviour of the D -function might have been interpreted as requiring non-perturbative power corrections from Eq.(7). While the addition of the twist-4 power corrections can be of real importance in the region of small enough Q^2 , deviation of the two-loop curves from the “experimental” corridor for the D -function at high momentum transfer (see Fig.1) can be associated with omission of perturbative QCD contributions. Another interesting observation is that the curves of Fig.1 and Fig.2 turned out to be rather smooth [16], lacking the resonance enhancements and “threshold steps”, typical of the Minkowskian region. In view of this we think that possible future applications and improvements of the results obtained in Ref.[16] can be useful for more detailed tests of perturbative and non-perturbative theoretical QCD predictions in the Euclidian region. One such application is presented in the next Section.

3. New tests of the generalized Crewther relation. The Crewther relation [42] connects the amplitude of $\pi^0 \rightarrow \gamma\gamma$ decay with the product of the e^+e^- annihilation D -function and deep-inelastic scattering sum rules, namely with Bjorken sum rule of polarized lepton-nucleon scattering

$$\begin{aligned} Bjp(Q^2) &= \frac{1}{6} \left| \frac{g_A}{g_V} \right| C_{Bjp}(Q^2) \\ &= \int_0^1 [g_1^{ep}(x, Q^2) - g_1^{en}(x, Q^2)] dx \end{aligned} \quad (11)$$

⁴For earlier discussion of the advantages of MOM-type schemes, taking account of threshold effects, see Ref.[49]

or with the Gross-Llewellyn Smith sum rule of νN deep-inelastic scattering

$$GLS(Q^2) = 3C_{GLS}(Q^2) = \int_0^1 F_3(x, Q^2) dx \quad (12)$$

where the analytical massless perturbative theory expression for the $D(Q^2)$ -function is known at the α_s^3 -level [29], while the massless analytical perturbative theory expressions for $C_{Bjp}(Q^2)$ and $C_{GLS}(Q^2)$ up to α_s^3 -corrections are known from the calculations of Ref.[47]. It should be stressed that all the quantities we will be interested in are defined in the Euclidian region and all the results were obtained in the \overline{MS} -scheme. In this scheme the α_s^3 generalization of the Crewther relation, discovered in Ref.[43], has the following form:

$$C_{Bjp}(\alpha_s(Q^2))C_D(\alpha_s(Q^2)) = 1 + \frac{\beta^{(2)}(\alpha_s)}{\alpha_s} P(\alpha_s) \quad (13)$$

where $C_D(\alpha_s(Q^2))$ is coefficient function for the Adler D -function, normalized to unity at lowest order, $\beta^{(2)}(\alpha_s)$ is the two-loop approximation of the QCD β -function and $P(\alpha_s)$ is a polynomial starts from α_s and contains two terms. It was suggested in Refs.[43, 50] that the factorization of the β -function will persist generally, to all perturbative orders, and can be related to the effects of violation of conformal symmetry by the renormalization of massless QCD. This was later proved in Ref.[51]. The theoretical properties of the generalized relation, written down in the \overline{MS} -scheme, were discussed in detail in Refs.[43, 45, 51] and we will avoid their description in this work. However, we will concentrate on some phenomenological applications. It is rather useful to use for this purpose commensurate scale relations [44], which combine the concept of effective charges [52] (or scheme-invariant perturbation theory [53]) with variants of the BLM approach [54, 55], allowing one to write the following generalization of the Crewther relation [41] in the region where the heavy-quark masses can be neglected:

$$\frac{1}{3 \sum_f Q_f^2} D(Q^2) C_{GLS}(Q_*^2) \approx 1 \quad (14)$$

$$\frac{1}{3 \sum_f Q_f^2} D(Q^2) C_{Bjp}(Q_*^2) \approx 1$$

where

$$\ln \frac{Q_*^2}{Q^2} = -\frac{7}{2} + 4\zeta_3 \quad (15)$$

$$+ \left(\frac{\alpha_{GLS}(Q_*^2)}{4\pi} \right) \left[\left(\frac{11}{12} + \frac{56}{3}\zeta_3 - 16\zeta_3^2 \right) \beta_0 - \frac{56}{27} - \frac{808}{9}\zeta_3 + \frac{320}{3}\zeta_5 \right]$$

As specimen physical input we will first use the recently obtained value of the Gross-Llewellyn Smith sum rule [56]

$$GLS(12.59 \text{ GeV}^2) = 2.80 \pm 0.13 \pm 0.17 \quad (16)$$

which does not contradict the less precise similar results, obtained using the extrapolation of the data [57]. Using the results of Ref.[58] we conclude that, in this energy region, $f = 4$ numbers of flavours are manifesting themselves. Moreover, at momentum transfer $Q_*^2 = 12.59 \text{ GeV}^2$ we will neglect c -quark mass effects, which are suppressed by a factor $m_c^2/Q_*^2 < 0.19$. Thus we conclude that the value of the corresponding effective charge is

$$\frac{\alpha_{GLS}(Q_*^2)}{\pi} = 1 - \frac{GLS(Q_*^2)}{3} \quad (17)$$

$$= 0.067 \pm 0.043 \pm 0.06$$

where the first (second) error is related to the statistical (systematical) uncertainty of the experimental result of Eq.(16). Using Eqs.(14)-(17) one can get the following estimate

$$D(Q \approx 1.8 \text{ GeV}) \approx 3.57 \pm 0.3 \quad (18)$$

which crosses the upper part of “experimentally” motivated curve of Fig.2 at this reference scale. A similar conclusion emerges in the case of analogous treatment of the experimental result of the SMC collaboration for the polarized Bjorken sum rule [59], which is

$$Bjp(Q^2 = 10 \text{ GeV}^2) = 0.195 \pm 0.029 \quad (19)$$

$$= \frac{1}{6} \left| \frac{g_A}{g_V} \right| \left(1 - \frac{\alpha_{Bjp}(Q_*^2)}{\pi} \right)$$

As a result of application of discussions in Ref.[60] and considerations presented above, we obtain the following estimate for the D -function

$$D(Q \approx 1.6 \text{ GeV}) \approx 3.59_{-0.45}^{+0.35} \quad (20)$$

which is in rough agreement with the “experimentally” motivated Euclidian results of Ref.[16], though with larger errors.

Of course, our considerations are not so detailed as the ones given in Ref.[61], where an expression for the value of the coupling constant α_s was extracted from the Bjorken sum rule data (for earlier discussions of this problem see Ref.[62]). However, we believe that even approximate tests of the generalized Crewther relation, written down in the form of commensurate scale relations of Ref.[41] **directly in the Euclidian region**, give one the feeling that estimates obtained from deep-inelastic sum rules are less precise than direct extraction of the behaviour of the D -function in the Euclidian region. This might suggest a need for adding higher-twist contributions to the perturbative generalization of the Crewther relation.

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REFERENCES

1. V.L.Auslender, G.I. Budker et al., *Phys.Lett.* **25B** (1967) 433.
2. J.E. Augustin et al., *Phys. Rev. Lett.* **33** (1974)1406.
3. J.J. Aubert et al., *Phys. Rev. Lett.* **33** (1974) 1404.
4. C. Bacci et al., *Phys. Rev. Lett.* **33** (1974) 1408.
5. P.N. Bogolyubov, N.V. Krasnikov, V.A. Kuzmin and K.G. Chetyrkin, *Fiz. Elem. Chast. Atom. Yadra* **7** (1976) 816.
6. M.I. Vysotsky, V.A. Novikov, L.B. Okun and A.N. Rozanov, *Phys. Usp.* **39** (1996) 503.
7. W.J. Marciano, hep-ph/9902332.
8. S.I. Eidelman and F. Jegerlehner, *Z. Phys.* **C67** (1995) 585; see also: H. Burkhardt and B. Pietrzyk, *Phys. Lett.* **B356** (1995) 398.
9. S.I. Eidelman and F. Jegerlehner, work in progress (1999).
10. F. Jegerlehner, preprint DESY 99-007 (hep-ph/9901386). and the review at this Workshop.
11. M. Davier and A. Höcker, *Phys. Lett.* **B419** (1998) 419; J.H. Kühn and M. Steinhauser, *Phys. Lett.* **B437** (1998) 425; J.G. Körner, A.A. Pivovarov and K. Schilcher, preprint MZ-TH-98-59 (1998) (hep-ph/9812465); M. Steinhauser, Report BUTP-99/06 (hep-ph/9904373); S. Groote, Report MZ-TH/99-18 (hep-ph/9905373).
12. S.G. Gorishny, A.L. Kataev and S.A. Larin, Proceedings of the Seminar “Physics of e^+e^- Interactions”, Dubna 8-9 December 1987; E2-88-363, p.33.
13. Talks at the Workshop “Bruno Toschek and the birth of e^+e^- physics”, Frascati, November 16, 1998.
14. S. Adler, *Phys. Rev.* **D10** (1973) 3714.
15. A. De Rujula and H. Georgi, *Phys. Rev.* **D13** (1976) 1296.
16. S. Eidelman, F. Jegerlehner, A.L. Kataev and O. Veretin, *Phys. Lett.* **B454** (1999) 369.
17. BES Collaboration, J.Z. Bai et al., hep-ex/9908046.
18. M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, *Nucl. Phys.* **B147** (1979) 385.
19. F. Jegerlehner, *Prog. Part. Nucl. Phys.* **71** (1991) 1, Sec. 4.5.
20. B.A. Kniehl, *Nucl. Phys.* **B347** (1990) 86.
21. S.G. Gorishny, A.L. Kataev and S.A. Larin, *Nuov. Cim.* **92A** (1986) 119.
22. K.G. Chetyrkin, R. Harlander, J.H. Kühn

- and M. Steinhauser, *Nucl. Phys.* **B503** (1997) 339.
23. K.G. Chetyrkin, J.H. Kühn and M. Steinhauser, *Nucl. Phys.* **B482** (1996) 213.
 24. J. Fleischer and O.V. Tarasov, *Z. Phys.* **C64** (1994) 413.
 25. K.G. Chetyrkin, S.G. Gorishny and V.P. Spiridonov, *Phys. Lett.* **160B** (1985) 149; V.P. Spiridonov and K.G. Chetyrkin, *Sov. J. Nucl. Phys.* **47** (1988) 522; L.R. Surguladze and F.V. Tkachov, *Theor. Math. Phys.* **75** (1988) 502; *Nucl. Phys.* **B331** (1990) 35.
 26. S.G. Generalis and D.J. Broadhurst, *Phys. Lett.* **139B** (1984) 85; D.J. Broadhurst and S.G. Generalis, *Phys. Lett.* **165B** (1985) 175; A.G. Grozin and Yu. F. Pinelis, *Phys. Lett.* **166B** (1986) 429; *Zeit. Phys.* **C33** (1987) 419; L.V. Lanin, V.P. Spiridonov and K.G. Chetyrkin, *Yad. Fiz.* **44** (1986) 1372.
 27. S.I. Eidelman, L.M. Kurdadze and A.I. Vainshtein, *Phys. Lett.* **82B** (1979) 278.
 28. A.G. Grozin and Yu. F. Pinelis, *Mod. Phys. Lett.* **A3** (1988) 725.
 29. S.G. Gorishny, A.L. Kataev and S.A. Larin, *Phys. Lett.* **B259** (1991) 144; *JETP Lett.* **53** (1991) 127; L.R. Surguladze and M.A. Samuel, *Phys. Rev. Lett.* **66** (1991) 560; *ibid.* 2416 (Err.); K.G. Chetyrkin, *Phys. Lett.* **B391** (1997) 402.
 30. V.A. Kuzmin, A.N. Tavkhelidze and K.G. Chetyrkin, *Pisma Zh. Eksp. Teor. Fiz.* **25** (1977) 456; K. G. Chetyrkin, N.V. Krasnikov and A.N. Tavkhelidze, *Phys. Lett.* **76B** (1978) 83.
 31. D.V. Shirkov and I.L. Solovtsov, talk at this Workshop (hep-ph/9906495); D.V. Shirkov and I.L. Solovtsov, *Theor. Math. Phys.* **120** (1999) 482 (hep-ph/9905305); *Dedicated to the memory of N.N. Bogoliubov.*
 32. B.V. Geshkenbein and B.L. Ioffe, hep-ph/9906406.
 33. D.V. Shirkov and I.L. Solovtsov, *Phys. Rev. Lett.* **79** (1997) 1209.
 34. Yu.L. Dokshitzer, G. Marchesini and B.R. Webber, *Nucl. Phys.* **B469** (1996) 93.
 35. A.C. Mattingly and P.M. Stevenson, *Phys. Rev.* **D49** (1994) 437.
 36. P.M. Stevenson, *Phys. Rev.* **D23** (1981) 2916.
 37. J. Chýla, A.L. Kataev and S.A. Larin, *Phys. Lett.* **B267** (1991) 269.
 38. H. Poggio, H. Quinn and S. Weinberg, *Phys. Rev.* **D13** (1976) 1958.
 39. OLYA/CMD Collab., L.M. Barkov et al., *Nucl. Phys.* **B256** (1985) 365.
 40. I.L. Solovtsov and D.V. Shirkov, *Phys. Lett.* **B442** (1998) 344.
 41. S.J. Brodsky, G.T. Gabadadze, A.L. Kataev and H.J. Lu, *Phys. Lett.* **B372** (1996) 133.
 42. R.J. Crewther, *Phys. Rev. Lett.* **28** (1972) 1421.
 43. D. J. Broadhurst and A.L. Kataev, *Phys. Lett.* **B315** (1993) 179.
 44. S.J. Brodsky and H.J. Lu, *Phys. Rev.* **D51** (1995) 3652.
 45. A.L. Kataev, Proc. II Workshop “Continuous Advances in QCD”, Minneapolis, March 1996, World Scientific 1996, p. 107 (hep-ph/9607426).
 46. S.J. Brodsky and J. Rathsmann, preprint SLAC-PUB-8176 (1999) (hep-ph/9906339).
 47. S.A. Larin and J.A.M. Vermaseren, *Phys. Lett.* **B259** (1991) 345.
 48. F. Jegerlehner and O.V. Tarasov, *Nucl. Phys.* **B549** (1999) 481.
 49. D.V. Shirkov, *Theor. Math. Phys.* **93** (1992) 1403; D.V. Shirkov and S.V. Mikhailov, *Z. Phys.* **C63** (1994) 463.
 50. G.T. Gabadadze and A.L. Kataev, *JETP Lett.* **61** (1995) 448;
 51. R.J. Crewther, *Phys. Lett.* **B397** (1997) 137.
 52. G. Grunberg, *Phys. Lett.* **B95** (1980) 70; *Phys. Rev.* **D29** (1984) 2315.
 53. A. Dhar and V. Gupta, *Phys. Rev.* **D29** (1984) 2822; V. Gupta, D.V. Shirkov and O.V. Tarasov, *Int. J. Mod. Phys.* **A6** (1991) 3381.
 54. S.J. Brodsky, G.P. Lepage and P.B. Mackenzie, *Phys. Rev.* **D28** (1983) 228.
 55. G. Grunberg and A.L. Kataev, *Phys. Lett.* **B279** (1992) 352.
 56. CCFR-NuTeV Collab., J.H. Kim, D.A. Harris et al., *Phys. Rev. Lett.* **81** (1998) 3595.
 57. A.L. Kataev and A.V. Sidorov, *Phys. Lett.* **B331** (1994) 179.
 58. J. Chýla and A.L. Kataev, *Phys. Lett.* **B297** (1992) 385.

- 59. SMC Collab., B. Adeva et al., *Phys. Lett.* **B412** (1997) 414.
- 60. K.A. Milton, I.L. Solovtsov and O.P. Solovtsova, *Phys. Lett.* **B439** (1998) 421.
- 61. G. Altarelli, R.D. Ball, S. Forte and G. Ridolfi, *Nucl. Phys.* **B496** (1997) 337.
- 62. J. Ellis, E. Gardi, M. Karliner and M.A. Samuel, *Phys. Rev.* **D54** (1996) 6986.